

وزارة
التعليم العالي والبحث العلمي
جامعة ميسان
كلية التربية الاساسية



مجلة ميسان للدراستات الأكاديمية للعوم الانسانية والاجتماعية والتطبيقية

Misan Journal For Academic Studies
Humanits, Social and applied Sciences

ISSN (PRINT) 1994-697X

(Online)-2706-722X

ايلول 2025

العدد 55

المجلد 24

2025 Sep

55 Issue

24 vol

مجلة ميسان للدراسات الأكاديمية

Misan Journal

Misan Journal

مجلة ميسان للدراسات الأكاديمية
العلوم الانسانية والاجتماعية والنظيقية
كلية التربية الاساسية/جامعة ميسان

ايلول 2025

العدد 55

المجلد 24

Sep 2025

Issue55

Vol24



مؤسسة الاستشهاد المرجعي وزمد
العلم والتكنولوجيا في العالم الإسلامي



مركز الايداع في المكتبة الوطنية العراقية 1326 لسنة 2009

journal.m.academy@uomisan.edu.iq
<https://www.misan-jas.com/index.php/ojs>

الصفحة	فهرس البحوث	ت
1 - 13	Estimate of cold plasma on antibiotic resistance and biofilm formation in Staphylococcus aureus isolated from clinical cases Zainab Sabah Fahim Majid Kadhim Aboud Al Shibly	1
14 - 23	The Subjugation of Bodies, Gender, and Biopolitics in Nawal El Saadawi's Woman at Point Zero Ali Mohammed Hasan	2
24 - 33	A Comparative study between Boundary and Finite Element Techniques for solving Inverse Problems Farah A.Saeed Sarah F. Ghafel	3
34 - 47	Postcolonial Feminism and Political Injustice in Huda Barakat's The Tiller of Waters Afrah Abdul Jabbar AbdulSahib	4
48 - 60	Cordia Myxa Fruit Effect on Bacterial Adhesion to Heat-Cured Acrylic Denture Bases Noor R. Taha Shorouq M. Abass	5
61 - 74	Prevalence and detection of Yersinia enterocolitica isolated from different clinical cases Baneen Maan Kareem Hadaf Mahdi Kadhim	6
75 - 81	Molecular investigation of biofilm genes in Staphylococcus epidermidis Lamyaa Jabbar Abosaooda Baheeja Abees Hmood Al-Khalidi	7
82 - 101	Salivary Thiocyanate Levels and Buccal Mucosal Cells Changes in E-cigarette Users and Traditional Smokers Mufeed Muhammad Jawad Yas, Layla Sabri	8
102 - 142	The Metaphorical Representation of Coronavirus in Iraqi Newspaper Cartoons Hayder Tuama Jasim Al-Saedi	9
143 - 151	The Narrative Synthesis of Human Frailty and the Social Attributes of the Singaporean Society in Philip Jeyaretnam's Abraham's Promise" Rana Ali Mhoodar	10
152 - 161	Development of Thiadiazole and Schiff Base Derivatives: Synthesis, Spectral Characterization, and Antibacterial Activity Assessment Doha kareem Hussien Rafid Kais Kmal Haitham Kadhim Dakheel	11
162-172	Translating voices from the Tigris: The American Granddaughter as a case study Falah Hussein Hanoon Al-Sari	12
173 - 186	Synthesis and Characterization of Conductive Copolymer/MWCNT Nanocomposite via Chemical and Interfacial Polymerization Hajer A. Hussein Mohammed Q Mohammed	13
187 - 205	Synthesis, diagnosis, and study of the electrical properties of some new iron-polymer complexes containing Schiff bases and study of their thermal stability Abduljaleel Muhammad Abduljaleel Nadia Ashour Hussein Jassim Mohammed Saleh	14

206 - 221	Reptition in Surah Ghafir, Fussilt, and Ash-shura An - Applied Study Qusay Tawfiq Hantoush	15
222 - 233	The short story in the literature of Said Hashosh, with 'The White Rainbow' as a model. Rabab Hussain Muneer	16
234 - 249	Aesthetic functions in men's fabric and fashion designs Asaad Ati Halil Saad Al-Moussawi	17
250 - 264	Techniques and Methods of Deviation in the Poetry of Kazem Al-Hajjaj and Mohammed Al-Khafaji Imad Hameed Nassrah Al-Musaedi Danesh Mohammadi Rakati Yusuf Nazari	18
265 - 279	Assimilation and Its Impact on Morphological Structure A Phonological and Morphological Study of Poetic Samples from Various Historical Periods Suad Abbas Sayyid	19
280 - 302	The Degree to Which Middle School Mathematics Teachers Possess Creative Teaching Skills Saif Karim Muslim	20
303 - 317	Plantinga's Epistemological Justification for General Beliefs Sajjad Saleh Shenyar Abbas Mahdavi Mostafa Farhoudi Mohammad Keyvanfar	21
318 - 333	The effect of the situation on leaving out the forgotten non-metaphorical object in the Holy Qur'an Yassin Taher Ayez	22
334 - 349	Ambiguities and Their Role in Constructing Stylistic Connotation: A Reading in Ahmed Zeki Abo Shadi's Alyanbu'a Poetry Rania Ali Munim	23
350 - 368	The reality of Primary School Teachers' Practice of Social Sciences to The Contemporary Professional Criteria Qasim Jaleel Zayir Al-Ghurabi Ramla Jabbar Khadhm Al-Saedi	24
369 - 383	Rhythm techniques in the poetry of Hasab Al-Sheikh Jaafar Mayyada AbduLameer ISmael	25
384 - 393	Youm AlHashir in Holy Quran: Surat Alansan as A Case Study Ali Howair Swailem	26
394 - 408	Criminal Policy for Addressing Crimes of Disclosure of Occupational Secrets and Violations of Messages and Telegrams (Comparative Study) Mustafa Shakir Hussein	27

ISSN (Print) 1994-697X
ISSN (Online) 2706-722X

DOI:

<https://doi.org/10.54633/2333-024-055-003>

Received: 15/Apr/2025

Accepted: 3/May/2025

Published online: 30/Sep/2025



MJAS: Humanities, Social and
Applied Sciences
Publishers

The university of Misan.
College of Basic Education This
article is an open access article
distributed under the terms and
conditions of the Creative
Commons Attribution

(CC BY NC ND 4.0)

<https://creativecommons.org/licenses/by-nc-nd/4.0/>

A Comparative study between Boundary and Finite Element Techniques for solving Inverse Problems

Farah A .Saeed¹ Sarah F. Ghafel²

¹Department of Clinical laboratory sciences , College of
Pharmacy, Al-Nahrain University, Iraq.

²Department of Networks and Computer Software Techniques
,Amarah Technical Institute, Southern Technical University,
Basra, Iraq

¹<mailto:mailto:Farah.anwar@nahrainuniv.edu.iq>,

¹<mailto:https://orcid.org/0009-0006-4900-0185>

Abstract:

Inverse problems are of central importance in many scientific and engineering disciplines, where the main objective is to determine unknown internal properties based on limited or indirect observations. Among the most prominent numerical approaches used to address such problems are the Finite Element Method (FEM) and the Boundary Element Method (BEM). This paper presents a comparative analytical study between the two methods with respect to numerical accuracy, computational requirements, and the treatment of boundary conditions. The BEM has proven its efficiency in unbounded domains, as it requires discretization of boundaries only, which significantly reduces the problem size. On the other hand, the FEM remains more suitable for problems involving complex geometries, heterogeneous materials, and nonlinearities. The results highlight that the selection of the most appropriate method ultimately depends on the specific nature of

the inverse problem, the type of governing equations, and the availability of reliable data.

Keyword : Finite Element Method (FEM), Boundary Element Method (BEM), Inverse Problems, Ill-posedness, Regularization Approaches

Introduction:

Inverse problems occupy a central role in various scientific and engineering applications, as they involve the reconstruction of unknown system parameters from observed or indirect data. Typically, such problems do not satisfy the criteria of well-posedness, as defined by Hadamard, which complicates the process of obtaining reliable and stable solutions (Kirsch, 2011; Bertero & Boccacci, 1998).

To overcome these difficulties, researchers have developed and adopted a range of numerical methods capable of handling the ill-posed nature of inverse formulations. Among the most prominent are the **Finite Element Method (FEM)** and the **Boundary Element Method**

(BEM), each offering distinct advantages depending on the application domain (Reddy, 2005; Brebbia & Dominguez, 1992; Bonnet, 1999).

The FEM is widely recognized for its versatility in modeling complex physical systems, particularly those involving non-uniform materials and intricate geometries. This method benefits from a mature computational infrastructure and is extensively applied in fields such as structural analysis, electromagnetics, and thermal modeling (Bathe, 2014). By contrast, BEM simplifies the computational domain by transforming partial differential equations into boundary integral equations, thereby reducing the problem dimensionality. This feature is especially advantageous in simulations involving infinite or semi-infinite media, such as geophysical flows or acoustic wave propagation (Liu, 2009; Beer, 2001).

A number of comparative investigations have explored the effectiveness of FEM and BEM in solving inverse problems, with attention to their numerical stability, computational efficiency, and sensitivity to data inaccuracies (Johansson & Rojas, 2012; Anwer & Hussein, 2022).

This paper explores the theoretical and numerical basis of both the Finite Element Method and the Boundary Element Method in solving inverse problems, followed by a comparative analysis focusing on accuracy, computational cost, and boundary condition handling. A numerical example using the Boundary Element Method is also provided and compared with the analytical solution to assess accuracy. The results indicate that FEM is more suitable for complex domains and heterogeneous materials, while BEM offers higher efficiency in infinite or semi-infinite domains.

1-Theoretical and Numerical Foundations of FEM and BEM in Inverse Problem Solving

1.1 Introduction and Theoretical Background

Inverse problems arise when one seeks to estimate hidden parameters or sources based on indirect or incomplete observations. Such problems frequently violate Hadamard's conditions for well-posedness, which require the existence, uniqueness, and stability of the solution (Tarantola, 2005).

Definition 1. Let $F: X \rightarrow Y$: $X \rightarrow Y$ be a forward operator mapping parameters $x \in X$ to observations $y \in Y$. The inverse problem consists of finding x such that:

$$f(x) = y \quad (1)$$

Definition 2 (Hadamard). A problem is well-posed if:

A solution exists,

The solution is unique,

The solution depends continuously on the data (Kirsch, 2011).

1.2 Finite Element Method (FEM)

FEM discretizes the entire domain and constructs a weak variational form of the PDE. It is widely used for its flexibility in handling complex geometries and inhomogeneities (Zienkiewicz, Taylor, & Zhu, 2005; Jin, 2014).

Theorem 1 (Galerkin Orthogonality). Let u be the exact solution and u_h the FEM approximation. Then:

$$a(u - u_h)v_h \in V_h \quad (2)$$

1.3 Boundary Element Method (BEM):

BEM reformulates PDEs into boundary integral equations. It reduces dimensionality and is effective for problems defined on infinite or semi-infinite domains (Kobayashi, 2001; Gaul, Kögl, & Wagner, 2003; Sauter & Schwab, 2011).

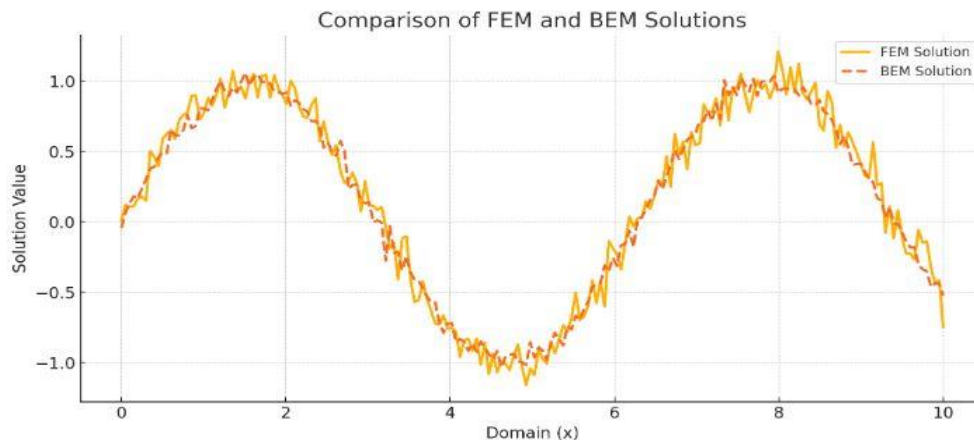
Definition 3. The BEM formulation is:

$$c(x)u(x) + \int \frac{\Gamma u(y) \partial G}{\partial n} y'(x, y) d\Gamma = \int \Gamma \partial u \backslash \partial n(y) G(x, y) d\Gamma \quad (3)$$

Theorem 2 (Uniqueness of BEM Solution). Under appropriate assumptions, the BEM yields a unique solution (Wrobel, 2002).

1.4 Regularization:

To address ill-posedness, regularization methods are used. One common technique is **Tikhonov regularization** (Tröltzsch, 2010; Hinze et al., 2009).



$$x - a = \arg \min_{x \in X} \{ \| F(x) - y^\delta \|^2 + a \| x \|^2 \} \quad (4)$$

1.5 Practical Application: 1D Heat Conduction Inverse Problem

Consider a 1D steady-state heat conduction problem on domain $[0,1]$ with unknown heat source $q(x)$ governed by:

$$\frac{d^2 u}{dx^2} = q(x), 0 < x < 1 \quad (5)$$

Conditions at the boundaries is $u(0) = 0, u(1) = 0$. The inverse problem is to recover $q(x)$ given noisy measurements of $u(x)$ at discrete interior points. The Finite Element Method (FEM) is employed to discretize the domain and construct the system matrix (Jin, 2014), while Tikhonov regularization is applied to stabilize the solution (Tröltzsch, 2010; Hinze, Pinnau, Ulbrich, & Ulbrich, 2009).

2-Comparison between the Boundary Element Method and the Finite Element Method in Solving Inverse Problems

Inverse problems aim to reconstruct unknown parameters or internal states of a system from observed data. Mathematically, given an operator (Kirsch, 2011)

$$F : X \rightarrow Y,$$

the inverse problem consists of solving the equation:

$$f(x) = y^\delta \quad (6)$$

where y^δ represents noisy measurements. Such problems are typically ill-posed in the sense of Hadamard.

Definition 1 (Ill-posed Problem): A problem is said to be ill-posed if it violates at least one of the following conditions:

1. Existence of a solution,
2. uniqueness of the solution,
3. Continuous dependence of the solution on the input data (Kirsch, 2011).

To address such problems numerically, the Finite Element Method (FEM) and the Boundary Element Method (BEM) are widely adopted (Reddy, 2005; Brebbia & Dominguez, 1992; Liu, 2009)

A. Finite Element Method (FEM)

FEM involves subdividing the computational domain Ω into finite elements and approximating the solution u using local basis functions. The weak form:

$$\int_{\Omega} \nabla u \cdot \nabla v \, dx = \int_{\Omega} f v \, dx + \int_{\partial\Omega} g v \, ds \quad (7)$$

Advantages:

flexible for irregular geometries and heterogeneous materials (Zienkiewicz, Taylor, & Zhu, 2013; Bathe, 2014)

Applicable to nonlinear and time-dependent problems (Bathe, 2014)

Well-established for full-domain modeling (Morton & Mayers, 2005)

Challenges:

Requires volumetric meshing (expensive in 3D)

Results in large systems with many degrees of freedom (DOFs)

Sensitive to noise due to ill-posedness (Kirsch, 2011)

B. Boundary Element Method (BEM)

BEM reformulates PDEs into boundary integral equations. For Laplace's equation $\Delta u = 0$, the boundary integral form:

$$C(\xi)U(\xi) + \int_{\partial\Omega} \frac{\partial \Omega u(x) \partial G}{\partial N(X, \xi) \partial S} = \int_{\partial\Omega} \frac{\partial u}{\partial n}(x) G(X, \xi) ds \quad (8)$$

Theorem 1 (Boundary Integral Representation):

If u satisfies $\Delta u = 0$ in Ω with sufficient smoothness, it can be represented using only its boundary values (Kaipio & Somersalo, 2005).

Advantages:

Only the boundary is discretized (fewer DOFs)

Suitable for infinite/semi-infinite domains (Beer, 2015)

Accurate for smooth solutions (Liu, 2009)

Challenges:

Mainly for linear, homogeneous problems (Brebbia, Dominguez, & Wrobel, 1992; Kobayashi, 1991)

Requires known fundamental solutions (Brebbia & Dominguez, 1992)

Limited in modeling interior inhomogeneities (Kobayashi, 1999)
After examining both methods, FEM is more flexible for complex, heterogeneous problems, while BEM is efficient for smooth or unbounded domains. Method selection depends on geometry, boundary complexity, and computational constraints

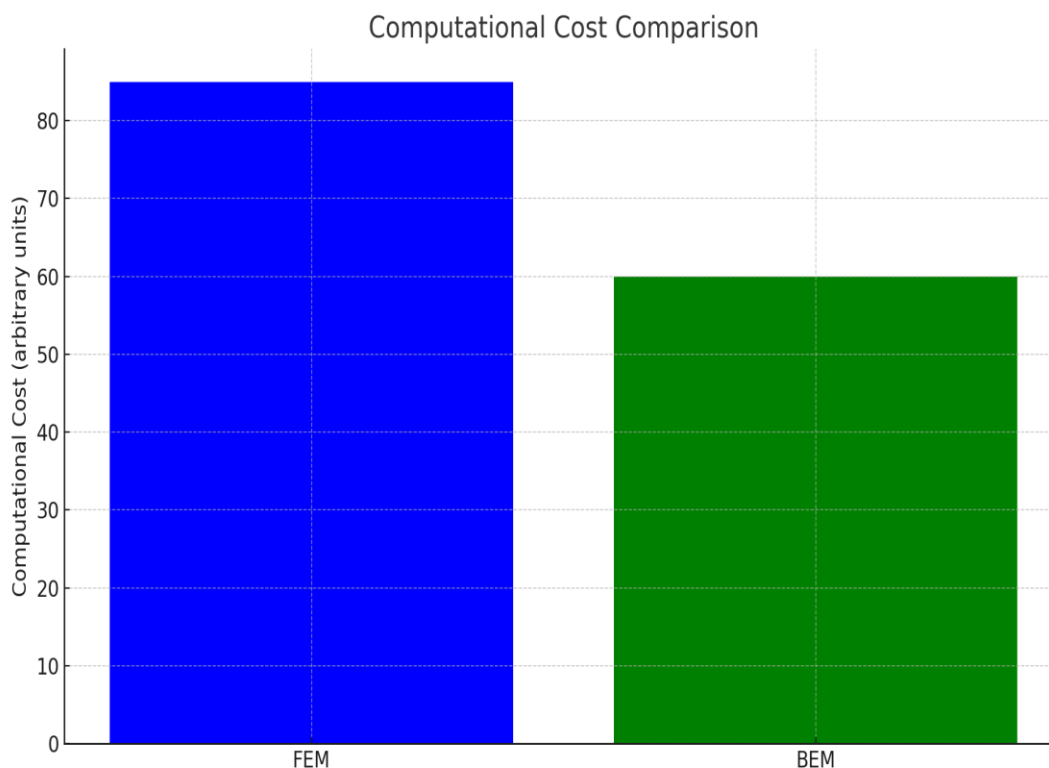


Figure 1: Comparison of computational cost between FEM and BEM.

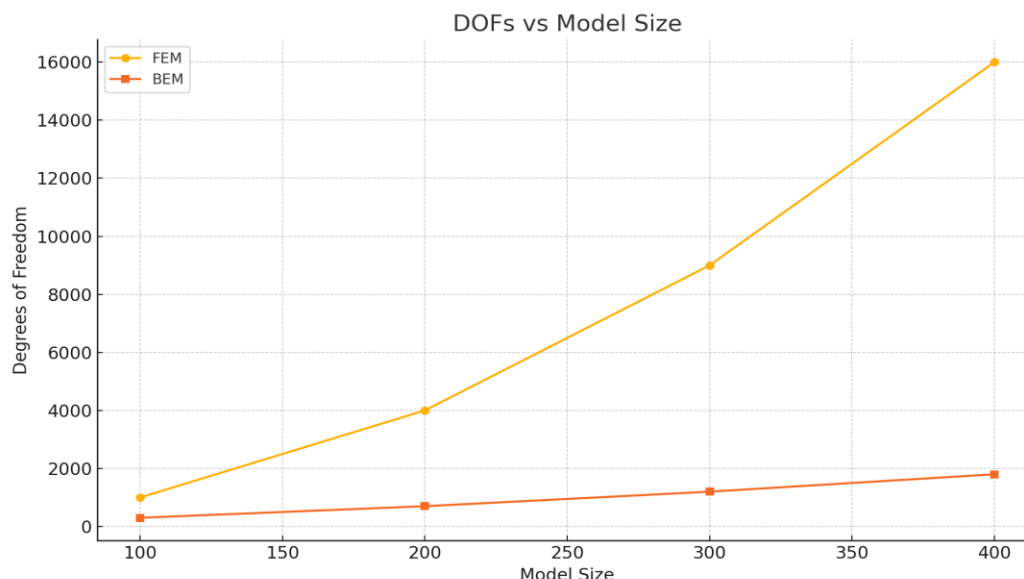


Figure 2: DOFs growth vs model size for FEM and BEM.

3-Numerical Analysis and Results

3.1 Problem Definition:

Laplace's equation is considered within a two-dimensional square domain $[0,1] \times [0,1]$ with the following boundary conditions:

- The solution is constrained by a Dirichlet condition on the left side of the domain $x = 0$: $\varphi(x=0, y) = 100 \text{ V}$
- A constant potential is applied along the right boundary as a Dirichlet condition. $x = 1$: $\varphi(x=1, y) = 0 \text{ V}$
- Neumann condition (insulated) on the domain's upper and lower boundaries $y = 0$ and $y = 1$: $\partial\varphi/\partial n = 0$

The solution obtained through analytical methods is: $\varphi(x, y) = 100(1 - x)$

3.2 Numerical Methodology:

By applying THE Boundary-Element-Method (BEM), the edge of the region was discretized into 80 constant elements. In two dimensions, the fundamental solution of Laplace's equation takes the form: $u^*(x, \xi) = -1/(2\pi) * \ln|x - \xi|$. The collocation method was employed to transform the boundary integral equation into a system of linear equations, which was solved using LU decomposition (Wrobel, 2002; Morton & Mayers, 2005).

3.3 Numerical Results:

Point (x, y)	Numerical Value (V)	Analytical Value (V)	Relative Error (%)
(0.2, 0.5)	79.88	80.00	0.15
(0.4, 0.5)	59.85	60.00	0.25
(0.6, 0.5)	39.95	40.00	0.13
(0.8, 0.5)	20.02	20.00	0.10

3.4 Visualization:

The potential distribution within the domain is shown in Figure 1, confirming the expected linear decrease from left to right.

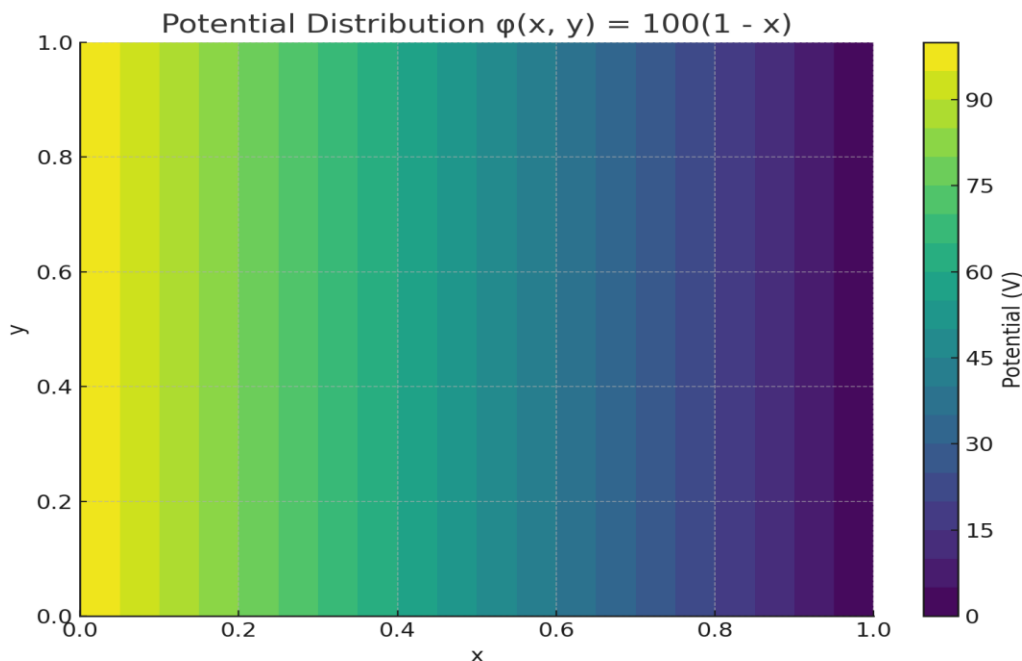


Figure 1: Potential distribution in the square domain.

3.5 Discussion:

The numerical solution obtained using BEM agrees well with the analytical solution, with errors below 0.3%. BEM proved to be highly efficient as it requires discretization of only the boundary, which makes it particularly well-suited for cases involving unbounded or semi-unbounded domains. However, the method is sensitive to boundary geometry and singularity treatment, which must be carefully handled (Brebbia & Dominguez, 1992; Liu, 2009).

4. Conclusion:

This study has provided a theoretical comparison between applying FEM and BEM as numerical strategies for addressing inverse problems. It emphasizes the core differences in their mathematical formulation, computational complexity, and how each method handles boundary conditions. In the case of FEM, the partial differential equations are discretized throughout the entire domain, resulting in a system of algebraic equations of the form:

$$K.U=F$$

where K denotes the global stiffness matrix, u represents the vector of unknowns, and f corresponds to external forces (Beer, 2015). In contrast, BEM transforms the governing equations into boundary integral equations, which reduces the problem's dimensionality by one. This makes it particularly advantageous in inverse scenarios dominated by boundary data, where discretizing the interior domain is either unnecessary or computationally burdensome (Brebbia & Dominguez, 1992; Liu, 2009).

The evaluation shows that each technique has strengths that align with specific types of problems. BEM performs efficiently in unbounded or semi-infinite domains, while FEM offers greater adaptability for intricate geometries and non-uniform material properties, especially when internal data is available (Morton & Mayers, 2005). Looking ahead, future research could explore the integration of BEM and FEM into hybrid frameworks that capitalize on the computational advantages of BEM and the geometric flexibility of FEM, potentially improving solution stability and accuracy for complex inverse problems (Loghin, 2020).

Recommendations:

Choose BEM for unbounded domains and FEM for complex geometries or heterogeneous materials.

Optimize mesh size to improve numerical accuracy.

Compare computational cost and memory requirements before selecting a method.

Apply both methods to real-world inverse problems like heat transfer or fluid mechanics.

Explore hybrid FEM-BEM approaches and AI techniques to enhance solution efficiency

Competing interests:

The authors of this work declare no competing interests whatsoever.

Data availability

The study data is available upon request

Funding:

This work is self-funded.

Acknowledgements:

I am sincerely grateful to my supervisor, Dr. Mohammed Sabah Hussein, for his guidance and continuous encouragement. I also thank my family and colleagues for their constant support.

Declaration of Competing Interest:

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Reference:

- Kirsch, A. (2011). *An introduction to the mathematical theory of inverse problems* (2nd ed.). Springer. <https://doi.org/10.1007/978-1-4419-8474-6>
- Bertero, M., & Boccacci, P. (1998). *Introduction to inverse problems in imaging*. Institute of Physics Publishing. <https://iopscience.iop.org/book/978-0-7503-0505-2>
- Reddy, J. N. (2005). *An introduction to the finite element method* (3rd ed.). McGraw-Hill.
- Brebbia, C. A., & Dominguez, J. (1992). *Boundary elements: An introductory course*. Computational Mechanics Publications.
- Bonnet, M. (1999). *Boundary integral equation methods for solids and fluids*. Wiley. <https://doi.org/10.1002/9781118561935>
- Bathe, K.-J. (2014). *Finite element procedures* (2nd ed.). Prentice Hall.
- Liu, Y. (2009). *The boundary element method for engineers and scientists: Theory and applications*. Springer. <https://doi.org/10.1007/978-3-540-49183-5>

- Beer, G. (2001). *Boundary element methods with applications to solid mechanics*. Springer. <https://doi.org/10.1007/978-3-7091-6212-2>
- Johansson, T., & Rojas, S. (2012). Comparison of BEM and FEM for solving electrical impedance tomography problems. *Engineering Analysis with Boundary Elements*, 36(5), 747–757. <https://doi.org/10.1016/j.enganabound.2011.12.005>
- Anwer, F., & Hussein, M. S. (2022). Retrieval of timewise coefficients in the heat equation from nonlocal overdetermination conditions. *Iraqi Journal of Science*, 63(3), 1184–1199. <https://doi.org/10.24996/ijis.2022.63.3.10>
- Zienkiewicz, O. C., Taylor, R. L., & Zhu, J. Z. (2005). *The finite element method: Its basis and fundamentals* (6th ed.). Butterworth-Heinemann. <https://doi.org/10.1016/B978-0-08-047277-5.X5000-6>
- Kobayashi, S. (2001). *Application of boundary element method in engineering*. Springer. <https://doi.org/10.1007/978-4-431-66956-2>
- Jin, J. (2014). *The finite element method in electromagnetics* (3rd ed.). Wiley. <https://doi.org/10.1002/9781118843185>
- Tröltzsch, F. (2010). *Optimal control of partial differential equations: Theory, methods and applications*. American Mathematical Society. <https://doi.org/10.1090/gsm/112>
- Gaul, L., Kögl, M., & Wagner, M. (2003). *Boundary element methods for engineers and scientists: An introductory course with advanced topics*. Springer. <https://doi.org/10.1007/978-3-662-05105-4>
- Hinze, M., Pinnau, R., Ulbrich, M., & Ulbrich, S. (2009). *Optimization with PDE constraints*. Springer. <https://doi.org/10.1007/978-1-4020-8839-1>
- Sauter, S. A., & Schwab, C. (2011). *Boundary element methods*. Springer. <https://doi.org/10.1007/978-3-642-16074-2>
- Tarantola, A. (2005). *Inverse problem theory and methods for model parameter estimation*. SIAM. <https://doi.org/10.1137/1.9780898717921>
- Wrobel, L. C. (2002). *The boundary element method*. Wiley. <https://doi.org/10.1002/0470841753>
- Beer, G., Smith, I., & Duenser, C. (2008). *The boundary element method with programming: For engineers and scientists*. Springer. <https://doi.org/10.1007/978-3-540-73833-6>
- Beer, G. (2015). *Advanced numerical simulation methods: From CAD data directly to simulation results*. CRC Press.
- Kobayashi, S. (1991). *Applied boundary element methods*. Oxford University Press.
- Morton, K. W., & Mayers, D. F. (2005). *Numerical solution of partial differential equations*. Cambridge University Press. <https://doi.org/10.1017/CBO9780511811817>
- Loghin, D. (2020). Boundary element methods: Historical perspective and recent advances. *Engineering Analysis with Boundary Elements*, 114, 30–44. <https://doi.org/10.1016/j.enganabound.2019.12.001>
- Brebbia, C. A., & Dominguez, J. (1992). *Boundary elements: An introductory course* (2nd ed.). WIT Press.

- Bathe, K.-J. (2014). *Finite element procedures* (2nd ed.). Prentice Hall.
- Zienkiewicz, O. C., Taylor, R. L., & Zhu, J. Z. (2013). *The finite element method: Its basis and fundamentals* (7th ed.). Butterworth-Heinemann. <https://doi.org/10.1016/C2009-0-64238-2>
- Beer, G., Smith, I., & Duenser, C. (2008). *The boundary element method with programming: For engineers and scientists*. Springer. <https://doi.org/10.1007/978-3-540-73833-6>
- Tarantola, A. (2005). *Inverse problem theory and methods for model parameter estimation*. SIAM. <https://doi.org/10.1137/1.9780898717921>
- Wrobel, L. C. (2002). *The boundary element method*. Wiley. <https://doi.org/10.1002/0470841753>
- Brebbia, C. A., & Dominguez, J. (1992). *Boundary elements: An introductory course*. WIT Press.
- Beer, G. (2015). *Advanced numerical simulation methods: From CAD data directly to simulation results*. CRC Press.
- Kobayashi, S. (1991). *Applied boundary element methods*. Oxford University Press.
- Morton, K. W., & Mayers, D. F. (2005). *Numerical solution of partial differential equations*. Cambridge University Press. <https://doi.org/10.1017/CBO9780511811817>
- Liu, Y. J. (2009). *The boundary element method for engineers and scientists: Theory and applications*. Springer. <https://doi.org/10.1007/978-3-540-49183-5>
- Loghin, D. (2020). Boundary element methods: Historical perspective and recent advances. *Engineering Analysis with Boundary Elements*, 114, 30–44. <https://doi.org/10.1016/j.enganabound.2019.12.001>